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Carbon Pricing and Global Warming: A Stock-flow Consistent Macro-dynamic Approach

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Abstract

To what extent can a worldwide carbon pricing foster the transition towards a low-carbon economy and help mitigate the effects of global warming? We address this question using a stock-flow consistent, financial and non-linear macrodynamics with uncertainty, calibrated for the world economy. We find that the upper-bound of the carbon pricing corridor advocated in the Stern-Stiglitz Commission, when implemented together with additional public subsidies on abatement costs in the private sector, succeeds in driving the economy into the neighbourhood of a balanced growth path. With high probability, this would make it possible to cap the average Earth temperature deviation at below +2.5°C by the end of this century. Absent such strong public involvement, and provided it be captured through a sufficiently convex damage function, the impact of climate change on gross output and capital appears to be powerful enough to almost surely pull the state of the world economy towards a debt-deflationary field, potentially leading to forced degrowth in the second half of the twenty-first century. Such a flow of trajectories is characterised on shorter time scales by low growth, the rise of unemployment as well as private debt, low inflation and interest rates, together with a declining wage share.

Key words: Ecological macroeconomics, Carbon pricing, Stock-flow consistency, Credit rationing, Lotka-Volterra

JEL Classification: C51, D72, E12, O13, Q51, Q54

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Carbon Pricing and Global Warming: A Stock-flow Consistent Macro-dynamic Approach

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ABSTRACT

To what extent can a worldwide carbon pricing foster the transition towards a low-carbon economy and help mitigate the effects of global warming? We address this question using a stock-flow consistent, financial and non-linear macrodynamics with uncertainty, calibrated for the world economy. More precisely, we assess the macroeconomic impact of carbon pricing and public subsidies by computing the probability densities of a large set of macroeconomic variables. Besides, we evaluate the extent to which such policies are sustainable, by computing the probability to remain below two thresholds that we argue to be critical for the stability of our current economy and climate: 1) a temperature anomaly above $+2°C$ (as set in the Paris Agreement) and 2) a global debt-to-output ratio. We find that the upper-bound of the carbon pricing corridor advocated in the High-Level Commission on Carbon Prices (2017), when implemented together with additional public subsidies on abatement costs in the private sector, succeeds in driving the economy into the neighbourhood of a balanced growth path. With high probability, this would make it possible to cap the average Earth temperature deviation at below $+2.5°C$ by the end of this century. Absent such strong public involvement, and provided it be captured through a sufficiently convex damage function, the impact of climate change on gross output and capital appears to be powerful enough to almost surely pull the state of the world economy towards a debt-deflationary field, potentially leading to forced degrowth in the second half of the twenty-first century. Such a flow of trajectories is characterised on shorter time scales by low growth, the rise of unemployment as well as private debt, low inflation and interest rates, together with a declining wage share.

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1 Introduction

The Paris Climate Agreement of December 2015 anchored the nationally determined contributions to the objective of remaining below $+2°$C. This diplomatic milestone has promoted carbon pricing as one of the key instruments to achieve these goals. Building on this momentum, the Stern-Stiglitz Report (High-Level Commission on Carbon Prices, 2017) recommended a worldwide corridor of carbon prices meant to be consistent with the Paris Agreement and the Sustainable Development Goals: from US$40-80/t$CO_2$ by 2020 to US$50-100/t$CO_2$ by 2030. Furthermore, the High-Level Commission on Carbon Prices (2017) also concluded on the necessity to adopt context-relevant policy packages in order to overcome the various barriers and failures associated with the carbon pricing instrument.

However, as forcefully stated by Erik Solheim, head of UN Environment, "current state pledges cover no more than a third of the emission reductions needed, creating a dangerous gap, which even growing momentum from non-state actors cannot close....That is why governments, private sector and civil society must bridge this catastrophic climate gap." (UNEP, 2017, p. xiii).

Based on these recommendations and Solheim's diagnosis, the aim of this paper is to assess the impact of the carbon corridor advocated by the High-Level Commission on Carbon Prices (2017) on economic growth and macroeconomic dynamics. Within this scope, the contribution of this paper is threefold. First, we provide an innovative framework to assess the economic consequences of carbon pricing by making use of a new hybrid financial macrodynamic model of the world economy. Within this approach, environmental feedbacks can be accounted for, as well as key variables such as inflation, underemployment, interest rate or private non-financial debt. We can then scrutinise how governments and the private sector could act together to achieve these goals.

Our second contribution sheds a new light on the macroeconomic consequences of implementing a worldwide carbon price within the corridor promoted by the High-Level Commission on Carbon Prices (2017). Furthermore, and as a follow-up to the Report's recommendations, we consider a second policy instrument, namely a public subsidy for mitigation technologies. Our broad conclusion is that the "climate gap" can be bridged with a reasonable probability of success *only* when carbon prices follow the upper-bound of the Stern-Stiglitz corridor and are complemented with significant public subsidies for the private sector's abatement efforts. This, of course, raises the question of the viability of the corresponding public debt dynamics. Yet, as it turns out, this issue can be managed under reasonable conditions. What's more, under circumstances that we carefully elucidate, the state of the economy will converge towards a long-run equilibrium equivalent to the steady state of the seminal Solow model. However, whenever these conditions are not met, our modelling approach does not preclude large economic imbalances—including the onset of deep recessions. In particular, absent strong public commitment, the impact of climate change on gross output and possibly on capital appears to be powerful enough to almost surely pull the state of the world economy towards a debt-deflationary field, potentially leading to forced degrowth in the second half of the twenty-first century. Such a flow of trajectories is characterised in the medium run by low growth, the rise of unemployment as well as private debt, low inflation and interest rates, together with a declining wage share.

Finally, our third contribution is to assess the extent to which the policy tools analysed in this article are able to avoid overshooting two thresholds that we argue are critical for the stability of our current economy and climate: 1) a temperature anomaly above $+2°$C (as set in the Paris Agreement) and 2) a global debt-to-output ratio above 270%.[1] Absent a commitment from the public sphere to subsidise as much as 30% of the private sector's abatement efforts, together with a carbon pricing equivalent to the upper-bound of the Stern-Stiglitz corridor advocated in the High-Level Commission on Carbon Prices (2017), our simulations suggest that we have no more than a 0.271% chance of achieving the $+2°$C warming target of the Paris Agreement and a 25.29% chance of the debt-to-income ratio staying below

[1] At which point the total private non-financial debt would exceed the value of the current stock of assets, arguably leading to systemic defaults.

the 270% threshold. Introducing climate policies as recommended by the Stern-Stiglitz Commission allows this probability to be increased significantly. Adding a growing carbon price trajectory increases the chances of staying under $+2°C$ warming to 5.786%. And adding a 30% subsidy for investment in the backstop technology to step up mitigation efforts increases this probability to 10.65%. We also discuss the objectives of keeping a sustainable debt ratio, transiting towards a low-carbon economy, and avoiding a recession. Effective climate policies do indeed come, in 2050, at the cost of increasing the median of the debt-to-output threshold from 1.5547 (business-as-usual scenario) to 1.7975 (with a carbon tax) and 1.7893 (with an additional subsidy for mitigation efforts).

These results hold as long as the damage function remains sufficiently convex.[2] At variance with other studies such as Nordhaus (2016), our approach aims to encompass relatively high temperature anomalies including "the prospect of large nonlinearities—and even abrupt degradation—of irreversible bifurcations toward wild trajectories", as envisioned in Henry and Tubiana (2017). We, therefore, consider not only Nordhaus (2016)'s damage function but also the (more convex) specification introduced by Dietz and Stern (2015), and compare their impact on the long-run viability of the world economy.

The main channel of the debt-deflationary gravitational attraction towards a global recession appears to be the interplay between damages inflicted by climate disturbances and private debt. Indeed, the former compels the productive sector to divert a growing part of investment to repairing degradation, which slows down output growth and abatement efforts towards a low-carbon economy, accelerating GHG loading and further aggravating global warming. At a certain point, either credit rationing (endogenously determined by the banking sector) or the intrinsic dynamic of private debt repayments brings about a debt overhang that may be detrimental to output growth and employment. A high carbon pricing helps to accelerate the transition as it provides additional incentives for the private sector to reduce GHG emissions. Obviously, however, it is of little help in defusing the vicious circle of damage/debt . Public subsidies then turn out to be the appropriate additional response[3] as they partially relieve the burden of abatement costs.

The article is organised as follows. Section 2 relates our work to the existing literature. Section 3 presents the accounting framework and defines the key equations. In Section 4, we discuss our various scenarios and the random variables underlying their dynamics. Section 5 compares these narratives and assesses the extent to which public policy can cope with climate change. Section 6 concludes.

2 Related literature

As we focus on the link between CO_2 loading, global warming and the dynamics of capital, debt, underemployment, inflation and world income, our work is very much in the spirit of environmental macroeconomics as advocated by Daly (1991) and ecological macroeconomics in the sense of Jackson (2009) (see also Victor and Rosenbluth (2014)). However, our modelling approach departs from these seminal contributions insofar as we adopt a stock-flow consistent[4] framework embodied in a low-

[2]We stress at the outset that we are well aware of the deficiencies of existing damage functions, see Dietz and Stern (2015); Hardt and O'Neill (2017). In several respects, choosing the right middle-ground between a disaggregated perspective, as embodied for instance in the World3 model of Meadows and Club of Rome and Potomac Associates (1972), and a highly aggregated but more tractable one, such as Nordhaus (2014), is still a controversial issue. Our first feeling, however, is that, given its policy implications, we cannot wait for better data before addressing the urgency of the "climate gap". Second, in our view, a strongly convex damage function is probably a more suited description of large-scale tipping points and thresholds such as a die-back of the Amazon rain-forest, a shift in monsoon systems, or the release of methane from marine hydrates (Knutti and Fisher, 2015). All these crucially need to be considered given the the span of temperature deviations examined in this paper.

[3]Obviously as one tool among others, which are not studied here, such as public-private partnerships, the use of public guarantees to minimise risk on private-sector balance sheets or even stranding some fossil-related assets.

[4]See Godley and Lavoie (2012) and, to quote just one illustration related to environmental economics, Godin (2012).

dimensional, continuous time, non-linear dynamical system where uncertainty is captured through a number of key climate and economic random variables. Stock-flow consistency emphasises the need for consistent accounting of all monetary stocks and flows as well as financial assets and liabilities. Non-linearity enables us to capture the consequences of large (as opposed to local) deviations from equilibrium or stationary situations.

The deterministic counterpart of our macro-dynamical system boils down to a 4-dimensional reduced-form that can be solved using analytical methods (see the Appendix A) and which belongs to the literature centered around Keen (1995).[5] The basic underlying dynamics is of the Lotka-Volterra type, which has already proven useful in order to account for endogenous environment-related cycles and breakdowns such as in Brander and Taylor (1998) and Motesharrei et al. (2014), to quote a few, and their possible impact on a stylised labour market as in Bernardo and D'Alessandro (2016).

On the strategic side, this modelling exercise seems to us, at this stage, to be a fruitful compromise between purely analytical approaches—which cannot cope with the high-dimensionality of real feedback loops at a macro-scale as identified by the physics of climate—and purely numerical approaches where the modeller is sometimes at risk of hardly understanding the behaviour of an *overly* large-scale model.[6] Of course, our own proposition has its own limitations, most of which are discussed in the concluding section.

In Campiglio (2016), an attempt is made to study carbon pricing and banking and monetary policy in relation to climate change. But its focus is on incentives that, beyond carbon pricing, might prevent commercial banks from shying away from lending for low-carbon activities. Here, the market failure in creating and allocating credit that we pinpoint is related to the firms' solvency. To the best of our knowledge, this is the first attempt to analyse carbon pricing (and beyond) within a stock-flow consistent (hereafter SFC) dynamical system calibrated for the world economy. In Jackson et al. (2014), the stock-flow consistent approach to macroeconomic modelling lends itself well, as it does here, to implementation in a system dynamics framework. However, the underlying macro-dynamic does not include non-neutral money, credit-rationing and private debt as we do. In addition, the authors do not consider carbon pricing.

The stock-flow consistent macrodynamic model presented in the next section is primarily borrowed from the world-scale economy with the climate interaction introduced in Bovari et al. (2017). We go beyond this previous work in four ways. First, we add a public sector that can roll out two types of public policy: a carbon tax and green subsidies to enhance the speed of the energy shift so as to act counter-cyclically against the debt-deflationary impact of climate change. Second, the banking sector's credit policy is endogenised along the lines suggested by Dafermos et al. (2017). Companies now face possible credit rationing when borrowing money from commercial banks—the most important source of external finance for firms—in order to finance investment. As a consequence, even though the accounting equation "investment = savings" of course still holds in our setting, it is no longer true that the increment of additional corporate private debt is always equal to households' current saving. Thus, money truly enters the picture.[7] Third, the short-run interest, which was exogenous, now follows a Taylor rule—which makes the dissipative friction of monetary policy itself endogenous. Fourth, three variables that were originally exogenous are replaced by random variables reflecting the scientific uncertainty surrounding some key variables of climate modelling, so that the outcome of our simulations consists of random trajectories. To be more specific, uncertainty is assumed for three main

[5] Such as Grasselli and Lima (2012),Grasselli et al. (2014), Nguyen-Huu and Costa-Lima (2014), Grasselli and Nguyen-Huu (2015) and Giraud and Grasselli (2017) *inter alia*.

[6] Analogous but different middle-grounds have already been proposed, such as the analytical system used by Bernardo and D'Alessandro (2016) for simulations, or the simple stock-flow consistent set-up of Berg et al. (2015), which can be analytically solved.

[7] As acknowledged by Pottier and Nguyen-Huu (2017) in previous related papers, the absence of any market failure on the banking side made the entire model compatible with a purely "real" interpretation, from which money would be absent.

drivers of the climate-economic system: (i) the magnitude of the long term-effect of climate change through the climate sensitivity parameter; (ii) the inertia of the climate system, which depends upon the size of the carbon reservoir; and (iii) the long-term growth pace of labor productivity.

Public policy, as captured here, remains rather simplistic, at least in comparison to its subtlety in Grasselli et al. (2014), where taxes and public spending are implemented in order to turn a non-desirable steady state into an unstable equilibrium—which thus has zero chance of being reached. One advantage of our more modest strategy is that it requires little knowledge about the world's economic peculiarities to be implemented.

In Nguyen-Huu and Costa-Lima (2014), the simple deterministic framework just portrayed *supra* was complicated by the introduction of Brownian motion, leading to a set of coupled stochastic differential equations. There, it was shown, however, that essentially the same dynamics prevails, albeit in a somewhat more fuzzy fashion. Here, we thus content ourselves with capturing uncertainty in a simpler way through, as already said, the randomness of a few key parameters.

3 A hybrid financial macro-dynamics

To assess the macroeconomic impact of climate policies, we introduce an innovative modelling framework that covers a wide range of economic situations without precluding the endogenous occurrence of large imbalances. Our starting point is an accounting framework introduced in Subsection 3.1 that structures the joint evolution of a macroeconomic module and a climate module, respectively introduced in Subsection 3.2 and Subsection 3.3.

3.1 The accounting framework

Let us start with the closed system of accounts shown in Table 1,[8] describing the balance-sheet, income statement and transaction flow matrices featuring all the monetary flows between the sectors of the economy. Balance sheet items are stocks measured in money, and both transaction items and the flow of funds items are flows measured in monetary units per unit of time.

The simplified economy is subdivided into households, firms, banks and an aggregate public sector. Their balance sheet structure can be described as follows: the assets of households are bank deposits, M_h, and equity, E, owned by shareholders; the assets of the private productive sector are bank deposits, M_f, and the stock of capital in nominal terms, pK; firms also have liabilities in the form of bank loans, L_c, and equity, E_f; banks have total deposits $M := M_h + M_f$ and equity, E_b, as their only liabilities, as well as loans, L_c, as their only assets. We therefore follow Bovari et al. (2017) and several preceding papers (see the references therein) by adopting the simplifying assumption that households do not take out bank loans.[9] The presence of equity as a balance sheet item implies that the net worth of firms is the value of corporate equity *plus* the difference between nominal capital, pK, and net nominal debt, D, to the banking sector. For simplicity, however, the net worth of banks and firms is kept identically at zero at all times. In particular, the corporate nominal debt, D, satisfies

$$\begin{aligned} D &:= L_c - M_c = pK - E_f \\ &= E_b + M_h. \end{aligned}$$

Say's law is assumed so that the model remains supply-driven.[10] Taxes and public subsidies are restricted to the firms' sector and limited to climate policies, since the purpose of this paper is to assess the macroeconomic impact of climate policies and their capacity to prevent severe climate-induced

[8]As usual, each entry represents a time-dependent quantity and a dot corresponds to time differentiation.
[9]This possibility can be added and would not qualitatively alter our results.
[10]Dropping this restriction, e.g., along the lines of Grasselli and Nguyen Huu (2018), is left for further research.

recessions. In line with Say's law, transfer payments to households and taxes from households would not play a significant role in the dynamics, and including them would not qualitatively affect our results.

	Households	Firms		Banks	Public Sector	Sum
Balance Sheet						
Capital stock		pK				pK
Deposits	M_h	M_c		$-M$		
Loans		$-L_c$		L_c		
Bonds	B				$-B$	
Equities	E	$-E_f$		$-E_b$		
Sum (net worth)	X_h	$X_f = 0$		$X_b = 0$	$-B$	X
Transactions		current	capital			
Consumption	$-pC$	pC				
Investment		pI	$-pI$			
Gov. Spend.		pG			$-pG$	
Acc. memo [GDP]		$[pY]$				
Wages	W	$-W$				
Capital depr.		$-\delta_{\mathbf{D}^{\mathrm{K}}} pK$	$\delta_{\mathbf{D}^{\mathrm{K}}} pK$			
Carbon taxes		$-pT_f$			pT_f	
Abatement subsidies		pS_f^C			$-pS_f^C$	
Int. on loans		$-r_c L_c$		$r_c L_c$		
Banks' dividends	Π_b			$-\Pi_b$		
Firms' dividends	Π_d	$-\Pi_d$				
Int. on deposits	$r_M M_h$	$r_M M_c$		$-r_M M$		
Int. on bonds	$r_B B$				$-r_B B$	
Column sum (balance)	S_h	Π_r	$-pI + \delta_{\mathbf{D}^{\mathrm{K}}} pK$	S_b	S_g	
Flow of Funds						
Change in capital stock		$p\dot{K}$				$p\dot{K}$
Change in deposits	$\dot{M}_h$	$\dot{M}_c$		$-\dot{M}$		
Change in loans		$-\dot{L}_c$		$\dot{L}_c$		
Change in bills	$\dot{B}$				$-\dot{B}$	
Column sum (savings)	S_h	Π_r		S_b	S_g	
Change in firm equity	$\dot{E}_f$	$-(\Pi_r + \dot{p}K)$				
Change in bank equity	$\dot{E}_b$			$-S_b$		
Change in net worth	$S_h + \dot{E}$	0		0	S_g	$\dot{p}K + p\dot{K}$

Table 1: Balance sheet, transactions, and flow of funds in the economy

Consistently with our assumed constant net worth of banks, the short-run interest rate, r, paid by firms to banks must be related to the return on deposit, r_M, paid to households, the dividends paid by banks to their shareholders, Π_b, and the change in equity of the banking sector

$$\dot{E}_b = rL_c - r_M M - \Pi_b.$$

We assume that banks issue or buy back shares accordingly. And similarly, for firms

$$\dot{E}_f = \Pi_r + \dot{p}K,$$

where, Π_r stands for the retained earnings of firms, K for the real stock of capital and p for an agreed price deflator. The flow of funds presented in Table 1 reflects the standard stock-flow consistency condition (Godley and Lavoie, 2012): financial balances for each sector are used to change their holdings of balance-sheet items. For example, central to the model (as in the literature initiated by Keen (1995)) is the fact that firms finance investment using both their financial balance and net borrowing from the banking sector according to the accounting identity

$$p\dot{K} - \Pi_r = \dot{L}_c - \dot{M}_c = \dot{D}.$$

3.2 The financial macro-dynamics

We now turn to the evolution of this macro-monetary accounting backbone by adding a continuous-time dynamics to its key entries.

3.2.1 Production and labour market

Absent climate change, we assume that firms can produce a potential real amount, Y^0, of a unique consumption good by combining the available workforce, N, and capital, K, with complementary factors of production

$$Y^0 \quad := \quad \min\left\{\frac{K}{\nu}; aN\right\}, \tag{1}$$

where $1/\nu > 0$ and a stand respectively for (constant) capital productivity and Harrod-neutral labor-augmenting progress.[11] Depending on the level of available capital, firms minimise their costs by hiring the required amount of labour at full capacity, $L := \frac{Y^0}{a} = \frac{K}{\nu}$, and the employment rate, λ, is endogenously given, $\lambda := L/N$. As defined shortly, industrial activities release CO_2-e emissions that are subject to a carbon tax levied by the public sector. In order to ease the tax burden, the productive sector may engage abatement activities so as to lower their emissions rate. Thus, a proportion, A, of output, Y^0, is removed from the commodity market and used as an intermediate consumption to reduce CO_2-e emissions. Moreover, as in Nordhaus (2016), a proportion, $\mathbf{D}^{\mathrm{Y}}$, of the remaining production is destroyed by global warming. As a result, the final production level is therefore

$$Y \quad := \quad (1 - \mathbf{D}^{\mathrm{Y}})(1 - A)Y^0. \tag{2}$$

The global workforce, N, is assumed to grow according to a sigmoid inferred from the 15–64 age group in the United Nations scenario

$$\dot{N} \quad := \quad \delta_N N \left(1 - \frac{N}{\overline{N}}\right), \tag{3}$$

where $\overline{N}$ is the upper limit of the global workforce and δ_N drives the convergence speed. Labour productivity is assumed to grow according to technical progress at an exogenous constant rate

$$\frac{\dot{a}}{a} \quad := \quad \alpha. \tag{4}$$

Finally, following Grasselli and Nguyen Huu (2018), the link between the real and nominal spheres of the economy is provided by two relationships. First, by a short-run Phillips curve[12] similar to the one introduced in Grasselli and Nguyen Huu (2018)

$$\frac{\dot{w}}{w} := \varphi(\lambda), \tag{5}$$

where w is money wage *per capita* and $\varphi(\cdot)$ will be an increasing real-valued function with a value in $[0; 1]$ calibrated. Second, denoting the consumption price as $p \geq 0$, an inflation dynamics will be

[11] The constancy of the capital-output ratio is in agreement with most of the post-Keynesian literature devoted to ecological macro-economics (see Hardt and O'Neill (2017)). Allowing for some substitutability between capital and labour, as well as putty-clay technology, is left for future research.

[12] See, e.g., Mankiw (2010) and Gordon (2014).

introduced, the latter relaxing with a speed, η, towards its long-run value, which is set as a constant markup, $\mu > 1$, times the unit labour cost, $\omega := wL/pY$, according to

$$i \;:=\; \frac{\dot{p}}{p} := \eta\left(\mu\omega - 1\right). \tag{6}$$

3.2.2 Emissions, taxation and abatement decisions

The nominal profit before dividends, Π, is defined as nominal output *minus* the cost of production: $\Pi := pY - wL - \delta_{\mathbf{D}^{\mathrm{K}}}pK - rD + p\Sigma$. Total cost is determined by: (i) the money wage bill, wL; (ii) the capital depreciation, $\delta_{\mathbf{D}^{\mathrm{K}}}pK$, with $\delta_{\mathbf{D}^{\mathrm{K}}} := \delta + \mathbf{D}^{\mathrm{K}}$, where $\delta > 0$ stands for the usual depreciation rate and $\mathbf{D}^{\mathrm{K}}$ for the fraction of capital destroyed by climate change (to be defined shortly); (iii) the debt service repayment, rD, with $r \geq 0$ as the short-run nominal interest rate and D the total nominal debt of firms; and (iv) the net public money transfers, Σ, to the productive sector, to be defined shortly. For simplicity, a fraction $\Delta(\omega, r, d) \in (0,1)$ of profit is paid to the households as dividends provided profits are non-negative. Consequently, the retained earnings of the corporate sector, Π_r, are given by $\Pi_r := \Pi - \Pi_d$, with $\Pi_d := \Delta(\omega, r, d)pK$. Depending on the level, p_C, of the carbon price (labelled in US\$2010 per t-CO_2-e), firms endogenously choose their emission reduction rate, $n \in (0,1)$. Industrial emissions, expressed in GtCO_2-e, are proportional to the potential production, Y^0, according to

$$E_{ind} := \sigma(1-n)Y^0, \tag{7}$$

where $\sigma > 0$ refers to the carbon intensity of the economy, which is assumed to follow some exogenous sigmoid function of time. The emission reduction rate, n, then obviously appears as the fraction of production processes that is “de-polluted”.

As in Nordhaus (2016), the abatement technology, A, is assumed to be a convex function of the emission reduction rate normalised by the emission intensity of the economy, σ, and the exogenous price of a backstop technology, p_{BS}, labelled in US\$2010 per t-CO_2-e,[13] following

$$A \;:=\; \frac{\sigma p_{BS}}{\theta} n^{\theta}, \tag{8}$$

where $\theta > 0$ controls the convexity of the cost.

Turning to the public sector, two instruments may foster the transition towards a zero-carbon economy. A carbon tax, $T_f^C := p_C E_{ind}$, may be levied on industrial emissions, E_{ind}, and a fraction, s_A, of abatement costs paid by firms, AY^0, may be subsidised by the public sector for a global transfer $S_f^C := s_A AY^0$. As a result, net transfers from the public to the private sector read

$$\Sigma := S_f^C - T_f^C. \tag{9}$$

Faced with the various policy instruments implemented by the public sector, firms choose the emission reduction rate, n, that minimises abatement costs *plus* carbon tax,

$$\min_{n\in[0,1]} AY^0 + T_f^C - S_f^C \;\text{ s.t. }\; \begin{cases} A &= \frac{\sigma p_{BS}}{\theta} n^{\theta} \\ T_f^C &= p_C\sigma(1-n)Y^0 \\ S_f^C &= s_A AY^0. \end{cases}$$

Consequently, the optimal aggregate abatement rate of emissions writes

$$n = \min\left\{\left(\frac{p_C}{(1-s_A)p_{BS}}\right)^{\frac{1}{\theta-1}} .1\right\}. \tag{10}$$

[13] For the sake of precision, p_{BS} grows at a constant (negative) rate, $\delta_{p_{BS}}$.

Notice that, as expected, the public sector's subsidies for abatement costs actually accelerate the optimal rate, n, of abatement efforts. From the perspective of firms, the price of the backstop technology is indeed lowered by the subsidies.

3.2.3 Investment and capital accumulation

Turning to investment, let us define the return on assets, π_K, by

$$\begin{aligned}\pi_K &:= \Pi/pK,\\ &= \frac{1}{\nu}\big((1-\mathbf{D}^{\mathrm{Y}})(1-A)\,(1-\omega-rd)-p_C\sigma(1-n)+s_AA\big)-\delta_{\mathbf{D}^{\mathrm{K}}},\end{aligned}$$

where $\omega := wL/(pY)$ is the wage-to-output ratio and $d := D/(pY)$ the debt-to-output ratio. Notice that the return on assets responds negatively to most of the variables related to climate change, namely A, $\mathbf{D}^{\mathrm{Y}}$ and p_c. On the other hand, the reduction of emissions will boost the profit rate.

Following Dafermos et al. (2017)'s insights, aggregate real demand for gross investment, I^D, is then driven by the return on assets, π_K, capturing the risk appetite of the productive sector

$$I^d \;:=\; \kappa(\omega, d, r)Y, \tag{11}$$

where $\kappa(.)$ is an increasing function (depending here on our state variables, ω, d and r) with a value in $[0;1]$ calibrated. Current profits may not suffice to finance the whole of I^d, in which case firms will have to borrow from the banking sector. Due to possibility of credit rationing, however, I^d might be only partially financed, as defined shortly. Let I refer to the real supply of investment $I := \Pi_r/p + \dot{D}/p + \delta_{\mathbf{D}^{\mathrm{K}}}K$, where $\dot{D}$ is the flow of credit granted by the banking sector. Capital accumulation then takes the standard form

$$\dot{K} := I - \delta_{\mathbf{D}^{\mathrm{K}}}K. \tag{12}$$

3.2.4 The banking sector

In order to finance gross investment and their various financial expenses, firms address an aggregate credit demand, D^d, to the banking sector

$$D^d := pI^d + s_{rep}D - \delta_{\mathbf{D}^{\mathrm{K}}}pK - \Pi_r,$$

where $s_{rep}D$ refers to the fraction of principal, D, that the productive sector has to pay back at each time instant to the loan holders (which, for simplicity are identified as being the banking sector).[14] As in Dafermos et al. (2017), the banks will satisfy this demand up to a credit rationing function, CR $:= \tau(\text{lev})$, where $\tau(\cdot)$ is an increasing function (with a value in $[0,1]$) of the leverage ratio, lev $:= \frac{D}{pK}$. It provides an upper-bound endogenously set by banks on the supply of additional credit to borrowers, according to their debt-to-capital ratio.[15] The aggregate nominal debt dynamics is therefore given by

$$\dot{D} \;:=\; (1-CR)D^d - s_{rep}D. \tag{13}$$

As a consequence, effective gross credit can be written as a fraction of available world income, Y:

$$\begin{aligned} I &= CR\big(\Pi_r/p + \delta_{\mathbf{D}^{\mathrm{K}}}K - s_{rep}D/p\big) + (1-CR)I^d \\ &=: \kappa_I(CR, \omega, \pi_K, \mathbf{D}^{\mathrm{K}}, \mathbf{D}^{\mathrm{Y}}, A, d, \lambda, r, \delta, s_{rep})Y. \end{aligned} \tag{14}$$

It can be readily seen from Eq. (14) that the function $\kappa_I(\cdot)$ generalises the investment behaviour first introduced, with no credit rationing, by Keen (1995).

[14] Making the yield curve and the schedule of capital repayments explicit would lead to a partial differential equation in an infinite-dimensional framework, which is left for further research.

[15] An alternative modelling option would consist in endogenising default and collateral constraints, following, e.g., Geanakoplos and Zame (2014).

3.2.5 Public sector and policies

The influence of the public sector is summarised by three variables: the real carbon price, p_C, the carbon-abatement subsidy rate, s_a, and the short term nominal interest rate, r. Each of these variables affects the profit share, hence the entire macro-dynamics through investment flows. For our baseline framework, we assume that the real price of carbon, p_C, grows exogenously at a given rate. For the purpose of our policy scenarios, this price will then be assumed to follow an exogenous path given by the High-Level Commission on Carbon Prices (2017). For simplicity, the subsidising part of state intervention (i.e., s_a) will also be assumed to be constant throughout. Finally, the short-term interest rate will follow a standard Taylor rule (Taylor, 1993)

$$r = \max\{0, r^* + i + \phi(i - i^*)\}, \tag{15}$$

where r^* is the long-term real interest rate, i^* the inflation rate commonly targeted by the monetary policy authority[16] and $\phi > 0$ a parameter that controls the magnitude of the central bank's response to inflation.

3.3 Introducing global warming and feedback

Climate change is modelled in a stylised way directly inspired by the DICE model of Nordhaus (2016), adapted here to our continuous-time setting. Both the macroeconomic and climate modules are then coupled through the damage function already introduced in Eq. (2).

3.3.1 The climate module

Global emissions, $E := E_{ind} + E_{land}$, are expressed in CO_2-e units and result from two sources: (i) industrial emissions, E_{ind}, defined in Eq. (7); and (ii) exogenous land-use emissions, E_{land}, decreasing at an exponential rate, $\delta_{E_{land}} < 0$, such that $\dot{E}_{land} := \delta_{E_{land}} E_{land}$. The emission intensity of the economy, σ, also follows an exogenous path given by $\dot{\sigma} := g_\sigma \sigma$ and $\dot{g_\sigma} := g_\sigma \delta_{g_\sigma}$, with $\delta_{g_\sigma} < 0$ a parameter.

The carbon cycle is described through a three-layer model that features the interactions between: (i) the atmosphere (layer AT), where emissions are released, (ii) the biosphere–upper ocean (layer UP) and (iii) the lower ocean (layer LO) the latter two acting as carbon sinks. Thus, in each layer $i \in \{AT, UP, LO\}$, the accumulation of ${CO_2}^i$ evolves according to

$$\begin{pmatrix} \dot{CO_2}^{AT} \\ \dot{CO_2}^{UP} \\ \dot{CO_2}^{LO} \end{pmatrix} := \begin{pmatrix} E \\ 0 \\ 0 \end{pmatrix} + \Phi \begin{pmatrix} CO_2^{AT} \\ CO_2^{UP} \\ CO_2^{LO} \end{pmatrix} \quad \text{with} \quad \Phi := \begin{pmatrix} -\phi_{12} & \phi_{12} C_{UP}^{AT} & 0 \\ \phi_{12} & -\phi_{12} C_{UP}^{AT} - \phi_{23} & \phi_{23} C_{LO}^{UP} \\ 0 & \phi_{23} & -\phi_{23} C_{LO}^{UP} \end{pmatrix}, \tag{16}$$

where $C_i^j := \frac{C_{j_{pind}}}{C_{i_{pind}}}, \; i, j \in \{AT, UP, LO\}$, with $C_{i_{pind}}$ denoting the pre-industrial CO_2-e concentration in the corresponding layer, i, and ϕ_{ij} standing for the diffusion coefficients between layers i and j.

The energy imbalance with regards to pre-industrial levels induces a radiative forcing in the atmospheric layer. The latter is composed of two terms: (i) an industrial forcing, $F_{ind} := \frac{F_{2\times CO_2}}{\log(2)} \ln\left(\frac{CO_2^{AT}}{C_{AT_{pind}}}\right)$, where $F_{2\times CO_2}$ stands for the forcing resulting from a doubling of the pre-industrial atmospheric concentration in CO_2-e and (ii) an exogenous radiative forcing, $\dot{F}_{exo}$, which grows linearly from its initial value to a plateau in 2100 as in Nordhaus (2016).

[16] Which might be thought of as the network of central banks.

The dynamics of temperature is given by a two-layer model describing the interplay between the atmosphere and upper ocean (resp. the lower ocean), with a mean temperature deviation, T (resp. T_0), with regards to the pre-industrial era

$$C\dot{T} := F - \rho T - \gamma^*(T - T_0), \tag{17}$$
$$C_0\dot{T}_0 := \gamma^*(T - T_0), \tag{18}$$

where ρ is the radiative feedback parameter, and γ^* is the heat exchange coefficient between the two layers. C and C_0, refer respectively to the heat capacity of the atmosphere, land surface and upper ocean layer, and to the heat capacity of the deep ocean layer. Observe that, within such a set-up, the equilibrium climate sensitivity (ECS) is defined by $S := F_{2\times CO_2}/\rho$.

3.4 Climate feedback-loop

In order to quantify the impact of global warming on the world economy, we again follow Nordhaus (2016) and adopt a damage function, $\mathbf{D}(\cdot)$, summarising the total economic feedback of environmental change:

$$\mathbf{D} := 1 - \frac{1}{1 + \pi_1 T + \pi_2 T^2 + \pi_3 T^{\zeta_3}}. \tag{19}$$

with $\pi_1, \pi_2, \pi_3, \zeta_3 \geq 0$. As argued by Stern (2013) and Dietz and Stern (2015), global warming may have an adverse repercussion not only on income but also on the factors of production themselves, such as capital, and especially on infrastructure. Following Dietz and Stern (2015), we therefore divide global damages, $\mathbf{D}(\cdot)$, into two components, one impacting the capital stock

$$\mathbf{D}^{\mathrm{K}} := f_K\,\mathbf{D}, \quad f_K \in (0;1), \tag{20}$$

while its complement affects the gross output, Y:

$$\mathbf{D}^{\mathrm{Y}} := 1 - \frac{1 - \mathbf{D}}{1 - \mathbf{D}^{\mathrm{K}}}. \tag{21}$$

4 Uncertainty and policy scenarios

Uncertainties on climate and economic growth are introduced in this section along with the different prospective rundowns that will frame our numerical analysis.

4.1 Climate and economic uncertainties

Uncertainty is made explicit for three main drivers of the climate-economic system: (i) the magnitude of the long-term effect of climate change captured through the climate sensitivity parameter, S;[17] (ii) the inertia of the climate system, which depends upon the size of the intermediate carbon reservoir, $C_{UP_{pind}}$; and (iii) the long-term growth rate of labour productivity, α.[18] The above parameters have been extensively discussed in the climate and integrated assessment literature.[19] Various estimates of

[17] S characterises the Earth's long-term, thermo-dynamic equilibrium global temperature response to a doubling of the pre-industrial CO_2 atmospheric concentration. Therefore, in the medium run, only probability estimates can be deduced from empirical observation.

[18] The allocation of climate impairments between output and capital will also be introduced in a subsequent section through the parameter f_K (see 20).

[19] See Knutti et al. (2017) and the references therein.

probability density functions (hereafter pdf) could thus be explored. In particular, and based on this literature, Nordhaus (2016) introduced pdfs for these parameters, assuming independent distributions as no information on their mutual dependence is available so far. We follow his approach and approximate the pdfs as illustrated by Figure 1).

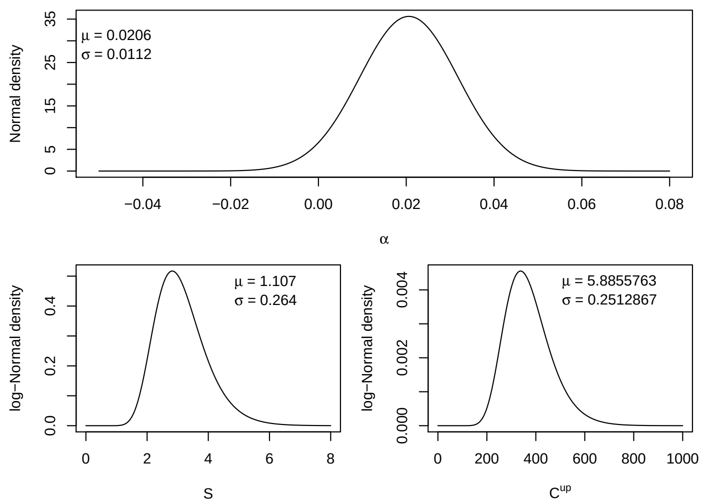

Figure 1: Probability density functions for the vector of parameters (α, S, C^{UP})

It is worth mentioning that Nordhaus (2016) considers two additional uncertain parameters in his sensitivity analysis: (i) the initial value of the decarbonisation rate of the economy, σ, and (ii) a coefficient of his damage function that captures the latter's curvature (or convexity), π_2. We refrain from modelling uncertainty for these two parameters for various reasons. For one thing, the academic consensus remains rather weak for the initial value of the decarbonisation rate of the economy as acknowledged by (Nordhaus, 2016). Moreover, the impact of this initial condition on the climate dynamics seems *prima facie* minor with regard to others such as the ECS or the size of the intermediate carbon reservoir which are considered here.[20] Besides, as the calibration of damage functions remains controversial (Dietz and Stern, 2015; Weitzman, 2011), we rely on two specifications of the damage function provided by the literature[21] and treat them as an additional parameter of our scenarios. At variance with Nordhaus (2016), we skip σ and π_2 in order to keep a reasonable dimensionality of the uncertain parameters so as to perform a *complete* Monte Carlo approach without relying on any approximation caused by excessive computational time.

4.1.1 Productivity growth

Nordhaus (2016) relies on estimates provided by a survey of experts performed by a team at Yale university led by Peter Christensen. Uncertainty on global output is characterised for the periods 2010–2050 and 2010–2100. Following these works, we adopt a Gaussian distribution for the labour productivity growth over the period 2016–2100 with a mean of 2.06% and a standard deviation of 1.12%: $\alpha \sim \mathcal{N}(0.0206, 0.0112)$.

[20]We performed some sensitivity analyses on extreme cases of the distribution chosen by Nordhaus (2016) for the initial level of the decarbonisation rate along with various sets of uncertain parameters. No major change in pattern emerged in our simulations: according to our set-up, the median of the pdf of the average temperature deviation is 3.93°C; as σ varies within the interval $[0.05, 0.95]$, the median final temperature rise varies within $[3.75, 4.16]$.

[21]See Nordhaus (2014) and Dietz and Stern (2015).

4.1.2 Equilibrium climate sensitivity (ECS)

The intrinsic uncertainty of the transient response of the climate system is captured by the ECS parameter.[22] We assume that the ECS follows a $\log$-Gaussian distribution with $\mu = 1.107$ and $\sigma = 0.264$. In other words, $S \sim \log -\mathcal{N}(1.107, 0.264)$—a choice motivated by the Bayesian estimates from Gillingham et al. (2015).

4.1.3 Carbon cycle

If many parameters of the carbon cycle are uncertain, the most important one remains the size of the intermediate reservoir (biosphere and upper level of the oceans), $C_{UP_{pind}}$ in our setup. This parameter will quantify the maximal upper level of the ocean capacity of CO_2 or, in other words, the inertia of the climate system. To take into account the uncertainty for this parameter, we follow the work of Nordhaus (2016), aimed at mimicking the results from Friedlingstein et al. (2014) in the difference of concentration in 2100 using the RCP8.5 CO_2 emissions. We thus calibrate a log-Gaussian distribution that is the closest to the quantiles reported by Nordhaus. In other words, $C_{UP_{pind}} \sim \log -\mathcal{N}(5.8855763, 0.2512867)$.

4.2 Climate risk

We finally consider the risks pervading the magnitude of climate damage to the economy and its channel of transmission. Indeed, as already said, climate change may impair output either directly or indirectly through various forms of harm to capital stock, which could in turn trigger potential contagion effects on the financial dynamics of the economy. We explore this channel by first testing two specifications of the aggregate damage function. The first is provided by Nordhaus (2016) and considered as a benchmark (labelled Mild) for our analysis as its validity is limited to a moderate range of global warming. However, as argued by Stern (2013), climate thresholds might be reached for higher ranges of temperature deviation, typically greater than $+4°$C—potentially giving rise to more severe and non-linear consequences. We thus also consider the more convex specification of the damage function introduced by Dietz and Stern (2015). Regarding the allocation of damages between output and capital, we consider the educated guess provided by Dietz and Stern (2015)'s interpretation of the results obtained byNordhaus and Boyer (2000), which are in the region of $1/3$. This means that capital stock sustains a third of the damages (in consumption-equivalent terms). We consequently test two different values: $f_K \in \{0, 1/3\}$ and take $f_K = 0$ as a reference point in order to facilitate comparison with earlier works such as Nordhaus (2016).

4.3 Public policy scenarios

First, we consider a *No feedback* scenario yielding a flow of trajectories absent any climate-change feedback loop. This scenario is then put in perspective with a *Mild feedback* scenario, introducing Nordhaus' specification for the damage function, along with a low real carbon tax growing at a constant 2% per year, in line with the Baseline scenario of Nordhaus (2014)). This second scenario is viewed as an intermediary step toward a baseline scenario incorporating climate feedback, and allows us to discuss the choice of a damage function. We finally introduce the three public policy scenarios considered in this paper: a scenario with a carbon tax calibrated from the High-Level Commission on Carbon Prices (2017) (labelled *Moderate public policy*), and a combination of the same carbon tax *plus* a subsidy for the backstop technology (called *Involved public policy*). The two corresponding narratives are con-

[22]For the calibration of the model, the heat capacity of the atmosphere, C, is updated according to climate sensitivity in order to adjust the TCR with the ECS as in Nordhaus and Sztorc (2013). Furthermore, carbon storage is precluded from the analysis.

trasted with a *Soft public policy* baseline where public intervention is again limited to a low real carbon tax growing at a constant 2% per year, in line with the Baseline scenario of Nordhaus (2014)).

As mentioned in the Introduction of this paper, theHigh-Level Commission on Carbon Prices (2017) has recommended a corridor of carbon price levels meant to be consistent with achieving the Paris temperature target and the Sustainable Development Goals: from at least US$40-80/t$CO_2$ by 2020 to US$50-100/t$CO_2$ by 2030.[23] In our simulations, we will focus on the upper barrier of the price corridor (hereafter High p_C). We stress that, as we take a long-term perspective, our simulations run from 2016 to 2100, while the recommendations of the High-Level Commission on Carbon Prices (2017) remain confined within the 2030 horizon in line with the Paris Agreement. Linear interpolations of the carbon price are therefore assumed to be outside the recommendation time range. Furthermore, as already said, we also consider public subsidies for mitigation technologies. The public sector subsidises the abatement cost as captured via Eq. (10). This subsidy could be interpreted as being funded by the carbon tax and thus viewed from firms' side as a price reduction on the abatement technology from p_{BS} to $(1 - s_a)p_{BS}$.

Table 2 summarises the five scenarios considered in the next subsection for our prospective analysis. Each of them is based on a complete Monte Carlo simulation, and will admit two variants depending on the chosen allocation of climate harm between capital and income, as described earlier by Eqs. (20) and (21).

Scenario	**No feedback**	**Mild feedback**	**Soft pol.**	**Moderate pol.**	**Involved pol.**
Low carbon tax		x	x		
High carbon tax				x	x
Abat. subsidy					x
Damage conv.		Nordhaus	Stern	Stern	Stern

Table 2: Scenarios considered for the prospective analysis

4.4 The dynamic landscape

Before plunging into our prospective analysis of the impact of a carbon price path in the next section, it is worth briefly describing the phase space of our dynamical economic system.

It can be shown[24] that, when debt financing investment together with a short-run Phillips curve are introduced, the global phase space of the standard macro-models inherited from the seminal works by Harrod, Domar, and Solow is significantly enriched. Indeed, the resulting deterministic non-linear dynamical system generally admits three long-run steady states, not just one. As one of these is locally unstable, it can be neglected without loss. A second long-term equilibrium—let us call it Solovian—turns out to be equivalent to the balanced-growth path in Solow (1956), albeit with different stability properties. It leads to a finite asymptotic private debt-ratio and nonzero wage share and employment rate. By contrast, a third steady state—the "explosive" one—is characterised by an infinite private debt-to-income ratio and collapsing wages and employment. Depending upon the basin of attraction to which the state of the world economy initially belongs, its subsequent path might differ sharply. Obviously, this dynamic landscape can be partially reshaped by climate feedbacks and climate policies. The former will tend to enlarge the basin of attraction of the explosive steady state, making it harder for the world economy to reach the safe Solovian equilibrium, whereas public policies, when properly designed, will aim to reinforce the attractive power of this equilibrium.

[23]No indication on the measure (real versus current) of the monetary unit is explicitly provided, even though some of the figures in the report are in 2005US$. We will therefore adopt the latter as the monetary unit of carbon pricing.

[24]See Keen (1995) and Bovari et al. (2017) for details.

Notice that, at variance with the Harrod-Domar knife-edge model, dynamic instability is not the only possible outcome here. But our approach also departs from the Solow-Swan setting as the Solovian equilibrium is not the only one where the state of the economy may stabilise.[25]

5 Prospective analysis

In this section, we present the main results of our prospective analysis on the macro-dynamic consequences of carbon pricing trajectories and public subsidies. First, we discuss the random paths followed by our key variables and, second, we examine their sustainability by computing the probability of overshooting two critical thresholds related to financial instability and the Earth's average temperature anomaly. The latter analysis allows us to quantify the macroeconomic impact of our policy scenarios.

Obviously, many criteria would deserve to be examined as critical lines. For simplicity, and with the hope of launching further inquiry on this issue, we discuss only two of them in this paper: (i) a 2°C temperature anomaly and (ii) a 270% total aggregate private debt ratio on output. In our view, they are informative on both climate and financial instabilities whose trade-off is the focus of this paper. The Paris Agreement set a 2°C threshold for temperature anomaly based on our current knowledge (gathered, e.g., by the IPCC[26]) on tipping points leading to severe and possibly uncontrolled damages to our economy and environment. On the other hand, our modelling approach also explicitly considers the financial channel as a cause of possible recession. At the level of an individual firm, whenever its liabilities exceed its assets, a company will arguably default on its debt (Geanakoplos and Zame (2014)). At the aggregate level, one can therefore consider that, if global private debt were to exceed the aggregate stock of capital, the global financial sphere would incur a serious risk of systemic defaults.[27] Using the Penn World Table (Feenstra et al., 2015), we calibrate the current global average capital-to-GDP ratio, ν, at 2.7. Since this ratio is constant in our modelling approach, it will play the role of the debt-to-output threshold we are looking for. As only part of our Monte Carlo simulations stay safely below these two specific ceilings, we shall compute the probability of overshooting them as an indicator of the underlying sustainability of the climate policies.

5.1 Is climate change really harmless in the long run?

Let us first compare the *No feedback* and *Mild feedback* scenarios. Figure 2 provides the outcome of the Monte Carlo simulations in the case $f_K = 0$ for the $[0.25, 0.75]$ probability interval of those scenarios, respectively in black and red shading (solid curves representing the point-wise median trajectories and dotted lines some comprehensive thresholds).

The first insight gained is that both scenarios exhibit very similar, if not entirely identical, patterns both in terms of median and support of the probability distribution of the trajectory flow. Real output grows close to the Solovian equilibrium following the (uncertain) exogenous labour productivity growth—the draw of the labour productivity growth, α, in the Monte Carlo procedure. The annual compound growth of the median GDP is roughly 2.38% over the whole period and 2.1% for the period 2050–2100 for both story lines.[28] Moreover, without climate feedback (*No feedback loop* scenario) or with a negligible carbon price (*Mild feedback* narrative), GDP growth still relies strongly on fossil

[25]Allowing for some substitutability between capital and labor would not suffice to recover global asymptotic stability of the Solovian steady state : it shrinks the basin of attraction of the "explosive" equilibrium, but does not make it disappear, as shown in Mc Isaac and Fabre (2018).

[26]See for instance Lenton et al. (2008), Stern (2013) or Carney (2015).

[27]Needless to say, this is a rough proxy as first-round effective bankruptcies would depend upon the various maturities of the contracted loans (together with their dependence on the yield curve), and second-round effects, upon the network of mutual dependences between firms and banks—these are important topics that go beyond the scope of this paper.

[28]Given the slow-down of population increase in the second half of the twenty-first century (as assessed by the United Nations median demographic scenario), long-run growth is entirely fuelled by a rise in labor productivity.

energy, and carbon neutrality is not reached in the twenty-first century. As a result, the median temperature anomaly in 2100 is estimated to be approximatively +3.96°C, almost twice the Paris Agreement's objective. Furthermore, both scenarios present a quite reasonable aggregate private debt of around 150% (of world income), an inflation rate that remains close to its commonly agreed target of 2%, and of course a positive (or null) climate primary public balance. Finally, in terms of distribution, the current trade-off between wages and capital would be hardly altered as the wage share would boldly remain in the region of 60% of world income. The only variable to follow distinct paths is the nominal short-run interest rate, which tends to be higher (in a $[0.25, 0.75]$ confidence interval) in the *No feedback loop* scenario than in the *Mild feedback* scenario. But this simply shows that a global monetary policy *à la* Taylor would have to be effective to keep inflation close to its current commonly agreed long-run target. Besides, in this case, monetary policy is sufficient to stabilise the world economy without any involved climate policy provided that climate feedback remains mild.

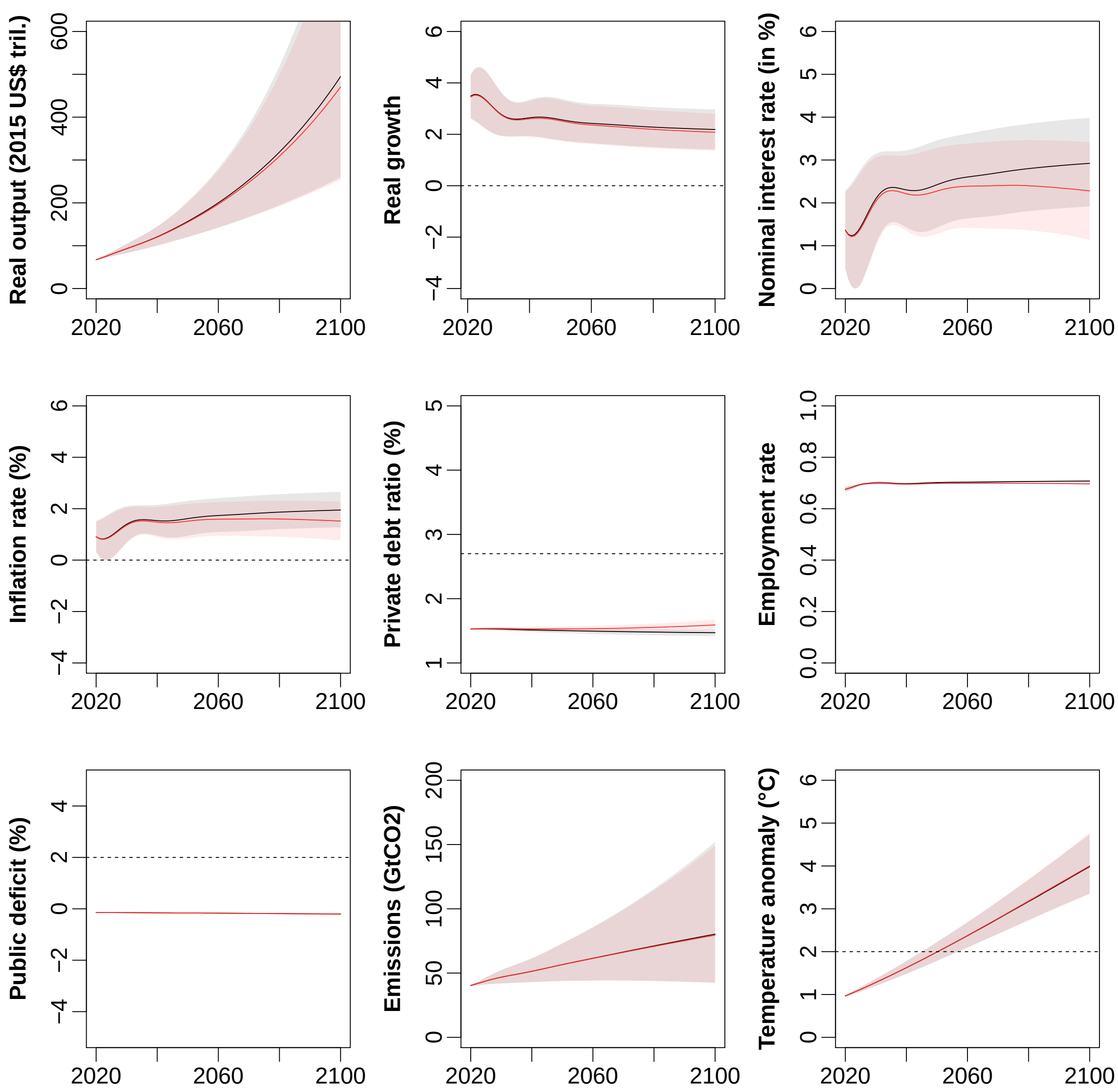

Figure 2: $[0.25; 0.75]$ probability interval of the **No feedback loop** and **Mild feedback** scenarios with a damage-to-capital ratio of 0 in black and red shades respectively (medians in solid curves).

As will be clear from the alternative story lines discussed in Subsection 5.2, the similarities between these two scenarios is essentially due to the choice of Nordhaus' damage function, whose feedback is too modest to affect the path followed by the world economy on a long-run basis. This should not be surprising since this damage function is calibrated for a rather small range of temperature anomaly, whereas global warming exceeds $+3°C$ by 2100 in both scenarios. The second insight from this comparative exercise is therefore that Nordhaus' specification for the damage function is probably seriously underestimated for the purposes of long-run prospective simulations that do not preclude large temperature anomalies. Despite cumulated CO_2 emissions, which in both narratives induce an average temperature rise of between +3.5 and +4.5°C (at a $[0.25, 0.75]$ confidence level), the real output is nearly identical with and without climate feedback. As documented in Stocker et al. (2013) and Knutti and Fisher (2015), this is hardly compatible with current estimates of the consequences of climate degradation at such levels of global warming.

5.2 Involved climate policies versus forced degrowth

We now turn, and for the rest of this paper, to the investigation of what might arguably be considered as more realistic narratives that incorporate a climate damage specification with a higher curvature as introduced by Dietz and Stern (2015). To what extent can the optimistic conclusions of our previous section be maintained? Figure 3 presents the $[0.25, 0.75]$ probability interval of the Monte Carlo simulations for the *Soft*, *Moderate* and *Involved policy* scenarios, respectively in red, orange and blue shades, in the case $f_K = 0$ (solid curves representing the point-wise median trajectory and dotted lines some comprehensive thresholds, discussed in the next subsection).

By contrast with the previous *Mild feedback* scenario, the *Soft policy* scenario no longer converges towards some balanced growth path: it instead exhibits a severe debt-deflationary pattern by the end of this century. Hence, the introduction of a more severe damage function (with unchanged policies) actually seems to foster the gravitational attraction of the "explosive" long-run steady state[29], calling for further action in order to mitigate climate change damages. The introduction of a carbon price as recommended by the High-Level Commission on Carbon Prices (2017) turns out to be sufficient for the world economy to withstand the attraction of the explosive basin, albeit not with absolute certainty, as illustrated by the *Moderate* and *Involved policy* schemes—whose outcome is roughly similar.[30] In both cases, the median real growth rate remains positive during the whole century and does not fall below 0.5% in the $[0.25, 0.75]$ probability interval. More precisely, compared to the *Soft policy* scenario, real GDP is 3.26% lower in 2050 in the *Moderate* case and 4.04% in the *Involved* case. The order is reversed in 2100, with a gain of 250% (resp. 199%) in the *Involved policy* narrative (resp. *Moderate policy* scenario). This is a direct consequence of an earlier implementation of the energy shift in both the *Moderate* and *Involved* cases through a carbon tax that triggers a fall of profits (and thus investment) and incentivises abatement activities, thus diverting a fraction of real output from sales. In any case, whenever the energy shift is implemented, global warming remains contained below $+3°C$, much lower damages occur at the end of the century, and the economy moves towards what seems to be a desirable long-run steady state. In particular, employment nearly reaches its benchmark value found in the *No feedback* scenario.

On the other hand, the *Moderate* and *Involved policy* narratives clearly depart from each other in terms of industrial CO_2 emissions and corporate non-financial debt ratios. Emissions in all simulations of the $[0.25, 0.75]$ probability interval reach the zero floor in 2084 in the *Involved policy* case against almost 2091 in the *Moderate* one. Furthermore, the private debt ratio does not exceed 270% in the $[0.25, 0.75]$ probability interval of the *Involved* scenario, while some simulations do in the *Moderate*

[29] Described in Subsection 4.4.

[30] For the sake of completeness, we also computed a variant of the *Moderate policy* scenario with the lower bound of the High-Level Commission on Carbon Prices (2017) carbon price corridor, which was shown to be insufficient to preclude severe climate-induced recessions. The corresponding figures are available upon request.

case—whereas the corresponding median is, of course, higher. The implications are twofold. First, the upper bound of the High-Level Commission on Carbon Prices (2017) carbon price corridor is apparently not sufficient to achieve the Paris Agreement's objective, whereas it is when backed with an abatement subsidy programme. Second—and this might have been counterintuitive at first glance—, the macroeconomic impact of more involved climate policies seems to be highly beneficial both in terms of climate risk (lower global warming in the median) and financial stability (lower private debt ratio). Lastly, the additional public current deficit induced by climate policy does not exceed 1.5% of GDP at its highest in 2085. It might obviously be offset by previous surpluses.[31]

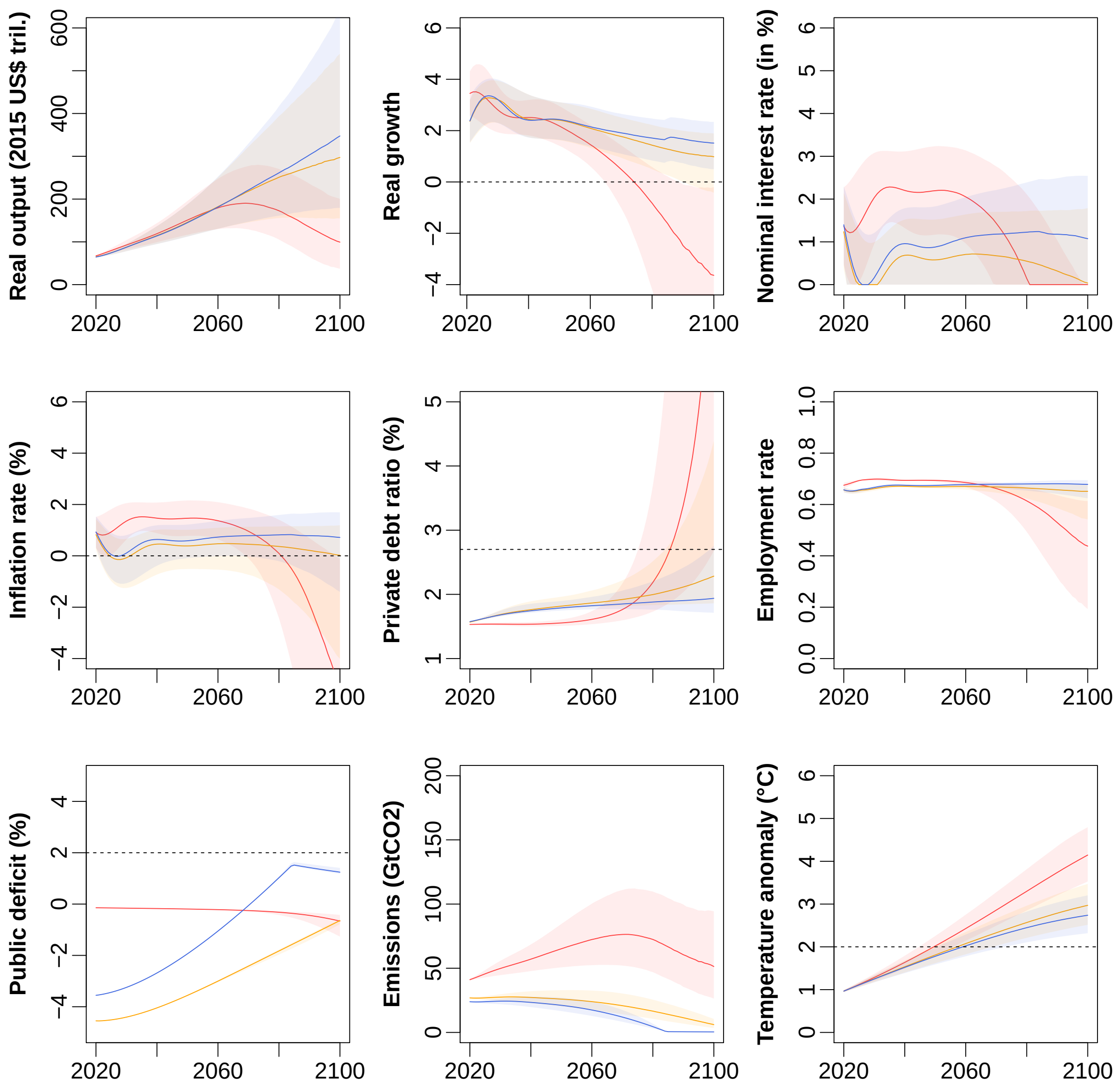

Figure 3: $[0.25; 0.75]$ probability interval of the *Soft*, *Moderate* and *Involved Policy* scenarios with a 0 damage-to-capital ratio, in red, orange and blue shades respectively (medians in solid curves)

The comparison of these various policy schemes also sheds new light on a climate-related awakening of the classical Keynesian trade-off between long-run inflation and employment. Indeed, the *Soft* scenario leads to a strong deflationary process where the median inflation rate becomes negative

[31] Public deficit, as reported here, is only the additional deficit (or surplus) induced by the environmental policymaking under scrutiny. Our calculation of the additional public climate debt burden does not exceed 4% of income.

shortly after 2080—and its worst case distribution tail even plunges below -4% during the last decade of the century—whereas underemployment reaches a hardly politically sustainable 60%. By contrast, the other two scenarios exhibit non-negative inflation rates throughout and employment rates that remain in the neighbourhood of today's 65%. In the best tail of the probability distribution of the *Involved policy* story line, inflation even remains close to the iconic 2%. Consequently, no analog of a vertical "long-run Phillips curve" can be observed in our setting. Hence, the structural long-run underemployment rate (around 35%), which still seems to hold in the vicinity of the Solovian steady state (towards which the *Involved* paths wander), should not be confused with the celebrated "natural rate of unemployment" introduced in Friedman (1968) and Phelps (1968). Moreover, the carbon price together with a monetary policy captured here through the setting of r do play a role, even in the long run, since they influence the asymptotic local stability of the balanced growth path. This contrasts with the very concept of "natural unemployment", which embodies the idea that any public policy whatsoever would be ineffective in reducing underemployment. Actually, our structural unemployment rate is closer to the NAIRU ("Non-Acccelerating Inflation Rate of Unemployment") introduced in Tobin (1980), since, at the balanced growth path, inflation stabilises.

5.3 Staying under reasonable climate-economic thresholds

A key lesson to be drawn from our previous findings is that public intervention is of paramount importance in order to escape from potentially harmful environmental feedbacks (whenever the latter is captured through a sufficiently convex damage function). Indeed, the debt-to-output ratio and temperature anomaly trajectories in Figure 3 suggest that only a fraction of the simulations makes it possible to stay safely below the two specific thresholds previously defined: (i) a 2°C temperature anomaly and (ii) a 2.7 debt-to-output ratio. In this section, we compute the probability distribution of the Earth's temperature rise and private debt and focus on the probability of overshooting these thresholds, which enlightens the possibility of climate-economic turmoil. Moreover, and in view of sensitivity analysis, we explore several allocations of damages between output and capital in the way introduced in Dietz and Stern (2015).

Scenario	**Variable**	$\mathbf{f_K = 0\%}$		
No feedback	*Quantile*	5	50	95
	$d > 2.7$ –	–	–	–
	$T > 2°C$	2073	2051	2040
Mild feedback	*Quantile*	5	50	95
	$d > 2.7$ –	–	–	–
	$T > 2°C$	2072	2051	2040
Soft policy	*Quantile*	5	50	95
	$d > 2.7$	–	2086	2065
	$T > 2°C$	2069	2050	2040
Moderate policy	*Quantile*	5	50	95
	$d > 2.7$	–	–	2061
	$T > 2°C$	–	2058	2042
Involved policy	*Quantile*	5	50	95
	$d > 2.7$	–	–	2066
	$T > 2°C$	–	2059	2043

Table 3: Dates at which thresholds are reached

The date at which a threshold is hit for a given quantile is given in Table 3. For instance, the 5$^{\text{th}}$

quantile of the random distribution of temperature rise induced by the *No feedback loop* scenario crosses the +2°C limit in 2073. As expected, for the 95^{th} quantile, the same cutoff point is reached no later than 2040. Three comments on Table 3 are in order: First, as expected, the deeper the public sector's involvement in fighting against climate change, the later the +2°C target is reached. For instance, only *Moderate* and *Involved policy* narratives make it possible to have more than a 5% chance of staying below the +2°C target until the end of this century. Second, the *No feedback* and *Mild feedback* scenarios show very similar dates, echoing our previous remarks. Third, for a sufficiently convex damage function, public policy involvement helps to delay the time at which the debt-to-output 2.7 deadline would be reached in the higher part of the probability distribution.

Next, we computed the probability distribution of the debt-to-output ratio and the temperature anomaly for the runs of all the parameter combinations in the three policy scenarios in 2050 and 2100. In Figure 4, the X-axis represents respectively the debt-to-output ratio and the temperature anomaly. The three policy schemes are stacked vertically, together with their two variants induced by the different damages-to-capital ratios considered ($f_K \in \{0\%, 33\%\}$) for each scenario. For each policy and each damage-to-capital ratio value, two distributions are represented. For the sake of clarity, only two policy scenarios are presented: *Moderate* and *Involved* policies. Indeed, as illustrated by Figure 3, the *Soft policy* scenario obviously exhibits both high indebtedness and high temperature anomaly probability densities. However, the *Mild feedback* scenario is displayed to facilitate the comparison with the remaining scenarios. The dark-blue-contoured distribution represents the frequency of all the runs in 2050, and the lighter contoured distribution, the same frequency in 2100.

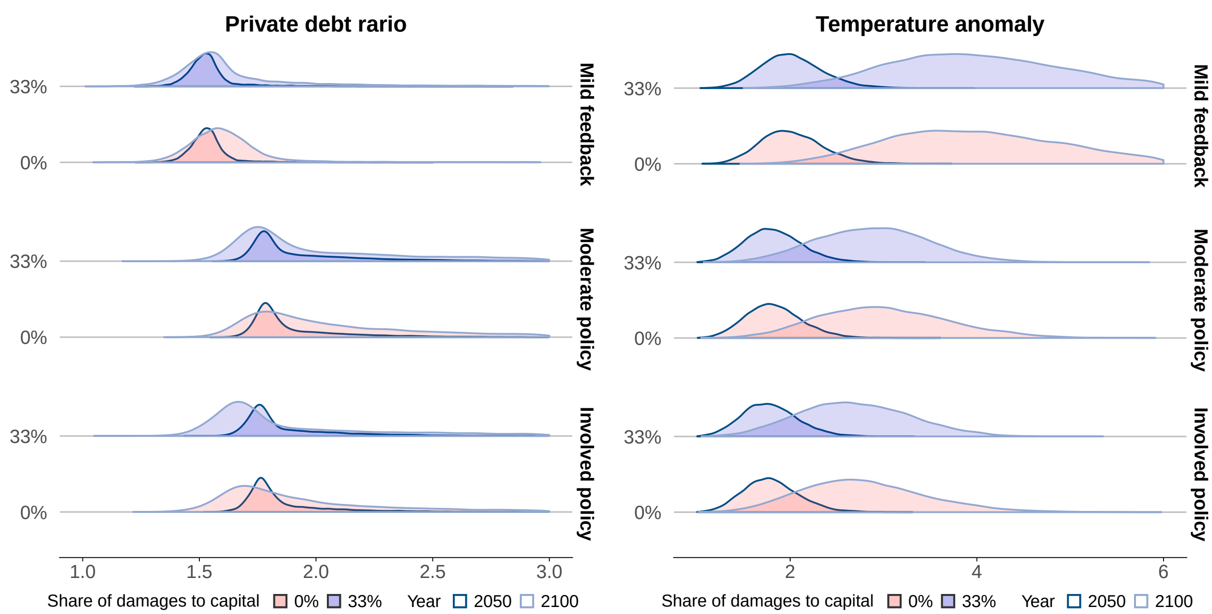

Figure 4: Probability density function as a function of the allocation of damages in 2050 and 2100.

The left-hand side of Figure 4 illustrates the effects on the debt-to-output ratio of an increasing fraction of damages allocated to capital, and the impacts of the policy mix. Increasing the damage-to-capital fraction pushes up the indebtedness of the private sector: it slightly shifts the pdf of the debt-to-output ratio to the right, leading to higher private debt relative to income around the thresholds under scrutiny. The phenomenon is similar in 2100, and can be observed in all the policy outlines, as shown by Table 4, which provides the survival function[32] of the debt-to-output ratio at the 2.7 level.

[32]In probability calculus, if $f(x)$ is the "survival (or reliability) function" of the random variable X at point x, then $f(x) = \mathbb{P}(X > x)$. This vocabulary might be slightly misleading since a higher survival function actually means that the state of the

$\mathbb{P}(d > 2.7)$ in %	2050	2100
No feedback loop	0.06	0.08
Soft feedback loop	0.00 (0.11)	0.57 (5.43)
Soft policy	0.08 (0.08)	74.71 (81.45)
Moderate policy	1.52 (2.54)	39.53 (58.01)
Involved policy	0.79 (1.35)	25.76 (44.15)

Table 4: Survival function of the debt-to-output ratio at 2.7 in 2050 and 2100 for $f_K = 0\%$. Values in parentheses are for the $f_K = 33\%$ whenever available.

Increasing the public climate policy involvement (from no policy to tax carbon emissions and supplementing the tax with a subsidy) also shifts the distribution to the left. This highlights the need for public intervention in balancing both financial stability and climate change, as will be discussed below. The right-hand side of Figure 4 displays the effect on the temperature rise of the share of damages induced by capital. As expected, the impact appears to be weaker than for the debt-to-output ratio. Alterations in the capital stock only marginally affect emissions—through the growth engine—and hence temperature anomaly. As shown in Appendix A, the profit-to-output ratio, π_K, decreases as damages, $\mathbf{D}^{\mathrm{K}}$, hit capital stock and thus investment. Consequently, private debt rises as, here, the decrease in investment is faster than the decrease in profits.

A comparison between scenarios in Figure 4 shows the benefits of public intervention to fight climate change and maintain financial stability. More effective climate policy mixes can be put in place with lower private debt in the long run. The underlying economic mechanism can be described as a kind of double-dividend. Indeed, when comparing *Soft policy* and *Moderate policy* scenarios, the carbon tax from the public sector provides stronger incentives to perform the energy shift and is thus effective in mitigating global warming. Consequently, the deep recessionary state showed by the *Soft policy* scenario is avoided, although, part of the debt-to-income ratio's pdf remains beyond the 270% threshold. By adding subsidy into the climate policy mix, the *Involved policy* scenario shows a more muted increase of private indebtedness than in the *Moderate policy* scenario. Indeed, transfers via subsidies increase firms' profit and thus investment. However, they also favour abatement activities and, then, real output in the longer term as damages are lower. The latter effect prevails given that the private debt ratio is reducing overall. These results are in line with the recommendations of the Stern-Stiglitz Commission (High-Level Commission on Carbon Prices, 2017), calling for a wider involvement of public actors, notably in terms of co-financing.

The impact of public policies is, however, prominent especially in the long run. As the *Involved policy* triggers more abatement efforts, it accelerates CO_2 emissions reduction. Table 5 shows that, from the *Soft policy* to the *Involved policy* cases, the median decrease of temperature anomaly is close to -1.4° C.

°C	$f_K = 0\%$	$f_K = 33\%$
No feedback loop	3.9857	—
Soft feedback loop	4.0023	3.989
Soft policy	4.1454	3.848
Moderate policy	2.9701	2.885
Involved policy	2.7387	2.668

Table 5: Median value of temperature anomaly distribution in 2100 reported in Figure 4

Lastly, Table 6 yields the probabilities of overshooting the temperature threshold of 2°C by measur-

world economy has *more* chances of being hit by severe imbalances.

ing the survival function at point. From the less successful sequences of events to the most successful one, we gain more than a 10% chance of achieving the 2°C target. In the central *Soft policy* scenario (with $f_K = 33\%$), there is less than a 1% chance of achieving the 2°C target by 2100, while this probability grows to above 6% in our *Moderate policy* scenario and exceeds 10% in our *Involved policy* scheme.

Prima facie, these figures might seem disappointing given the strong political momentum around the Paris Agreement. They are nevertheless comparable to the figures recently presented by Raftery et al. (2017) using a completely different methodology. The authors run a projection based on Kaya's identity at the country level. GDP and CO_2 emissions are forecasted in a probabilistic way, so as to compute the chances of staying below the 2°C line. They also find a 5% chance of meeting the Paris Agreement. This is consistent with our *Moderate policy* scenario, implying that minimal public intervention in favour of abatement efforts is required to achieve this result. Contrary to this paper, however, Raftery et al. (2017) propose no way of disentangling the different channels at play, nor do they estimate the effect of global climate policy mixes. We find that increasing the stringency of the policy mix by adding a subsidy for abatement technology allows the probability of achieving the Paris Agreement's target to be raised to 10%.

%	$\mathbb{P}(T > 1.5)$ in 2050	$\mathbb{P}(T > 2)$ in 2100
No feedback loop	95.397	99.483
Soft feedback loop	95.371 (95.496)	99.59 (99.496)
Soft policy	96.098 (96.206)	99.729 (99.747)
Moderate policy	86.78 (86.41)	94.214 (93.492)
Involved policy	84.39 (83.59)	89.35 (88.03)

Table 6: The survival function for the temperature anomaly at the 1.5°C in 2050 and 2°C 2100 for $f_K = 0\%$. Values in parentheses are for the $f_K = 33\%$ whenever available.

6 Conclusion

In this paper, we introduce an innovative modelling framework in order to assess the macroeconomic impact of implementing carbon pricing. The new modelling approach introduced in this paper reveals some striking facts: as soon as environmental feedbacks are captured through a sufficiently convex damage function, endogenous market forces do not seem strong enough to prevent the state of the world economy from being almost certainly ensnared by the attraction of a recessionary long-run steady state (possibly reached in the twenty-second century, while forced worldwide degrowth could begin, with high probability, as early as in the second half of this century). In the medium run, the trajectories leading to such a recessive state are characterised by decelerating growth, increasing unemployment, skyrocketing private debt,[33] a declining wage share, rapidly growing greenhouse gas emissions as well as a strong deflationary pressure on consumption prices and interest rates. The world economy is currently experiencing some of these alerting stylised features.

Fortunately, worldwide public involvement turns out to be successful in driving the global economy away from the dangerous "neighborhood" (within the phase space) of such a global recession. Indeed, we found that a worldwide carbon price following the upper-bound of the corridor advocated in the High-Level Commission on Carbon Prices (2017) would be enough to push the median trajectories of the world economy towards the basin of attraction of a Solovian long-term equilibrium. This, however,

[33] As already said regarding public finances, our simulations only concern the possible extra deficits entailed by public spending and taxes related to climate changes. The intrinsic evolution of non-climate-related public debt is left for future research as it requires a more detailed description of public interventions.

is compatible with the two following features: first, some worst-case distribution tails of our simulations, where high levels of private non-financial debt are possibly reached and, second, some medium-run trade-off in terms of lower real income growth and higher non-financial indebtedness.. However, the economy appears to be better-off at the end of the century due to the containment of climate damages. Moreover, prudent climate risk management suggests the need to ask for additional policy tools: in this paper, we show that, whenever abatement costs are sufficiently subsidised in addition to the implementation of some carbon pricing, even the worst-case distribution tails of our simulations seem to become viable.

This paper calls for a number of extensions that have already been partly suggested throughout the paper.[34] Firstly, designing the financial system is a key challenge to elucidate its complex interaction with the consequences of climate change (Jackson et al., 2014). We view our paper as a first attempt to tackle this issue. A more detailed description of the prudential rules followed by the banking sector (e.g. following Giraud and Kockerols (2015)), or introducing endogenous default and collateral constraints (e.g. following Geanakoplos and Zame (2014)) would make it possible not only to better understand credit rationing but also to propose some recommendations for macro-prudential regulation. Furthermore, numerous scholars have pointed out the key issue linking climate change and inequality (IPCC, 2014; Jackson et al., 2014). In this paper, inequality has only been dealt with in terms of the classical trade-off between capital return and wages: as climate destruction seems to boost the share of capital within an otherwise possibly decelerating (or even declining) world income, the higher the wage share, the more resilient the economy. A deeper grasp of the issue at stake could obviously be obtained by disaggregating households (e.g. following Giraud and Grasselli (2017), who suggest, without climate considerations, that reducing inequality may foster growth under some specific circumstances). Moreover, our system dynamical approach is well suited to dealing with other key issues related to the energy shift, such as the depletion of fossil fuels and mineral stocks:[35] Capturing how the latter can feed back to affect economic performances would have considerable consequences for policymaking and will be considered in future research. Lastly and most importantly, there is a need to explore other public policy instruments, such as subsidies for green capital specifically or differentiated interest rates for loans to stimulate green investments. These might lower the burden of the carbon tax on the private sector and ease the transition towards a low-carbon economy.

Acknowledgements

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Appendices

A Reduced-form

The climate-economy interplay in our model can be wrapped up through a simple reduced-form. Given the trajectories of the climate-side of the model, namely $(\mathbf{D}^{\mathrm{Y}}, \mathbf{D}^{\mathrm{K}}, A, T, p_c, p_{BS})$, the dynamical system

[34] E.g., in Footnotes 10, 11, 14, 15 and 33.

[35] A Lotka-Volterra perspective proved useful, e.g., in the fish-stock model by Cordier et al. (2017). On the other hand, Vidal et al. (2017) have shown that a number of minerals—first and foremost, copper—will presumably reach their extraction peak at a planetary scale in the coming decades.

boils down to a 4-dimensional system:

$$\begin{cases} \frac{\dot{\omega}}{\omega} &= \phi(\lambda) + (m-1)i - \frac{\dot{a}}{a} + \frac{\dot{\mathbf{D}^{\mathrm{Y}}}}{(1-\mathbf{D}^{\mathrm{Y}})} + \frac{\dot{A}}{(1-A)} \\ \frac{\dot{\lambda}}{\lambda} &= \frac{1}{\nu}\kappa_I(\cdots)(1-A)(1-\mathbf{D}^{\mathrm{Y}}) - \delta_{\mathbf{D}^{\mathrm{K}}} - \alpha - \frac{\dot{N}}{N} \\ \frac{\dot{d}}{d} &= \frac{1}{d}\left(\kappa_I(\ldots) - \frac{(\pi_K - C_{\mathcal{P}} - \delta_{\mathbf{D}^{\mathrm{K}}} - \mathbf{D}^{\mathrm{K}})\nu}{(1-A)(1-\mathbf{D}^{\mathrm{Y}})} + \Delta(\omega, r, d)\right) - i - \frac{\kappa_I(\ldots)}{\nu}(1-A)(1-\mathbf{D}^{\mathrm{Y}}) + \delta_{\mathbf{D}^{\mathrm{K}}} + \frac{\dot{A}}{(1-A)} + \frac{\dot{\mathbf{D}^{\mathrm{Y}}}}{(1-\mathbf{D}^{\mathrm{Y}})} \\ \dot{N} &= \delta_N N \left(1 - \frac{N}{\overline{N}}\right) \end{cases}$$

with

- $i = \frac{\dot{p}}{p} := \eta\left(\mu\omega - 1\right)$
- $\pi_K = \frac{(1-\mathbf{D}^{\mathrm{Y}})(1-A)}{\nu}\left(1 - \omega - rd\right) - \delta_{\mathbf{D}^{\mathrm{K}}} - p_C \frac{\sigma(1-n)}{\nu} + \frac{s_A A}{\nu}$
- $\kappa_I(\ldots) = \left[CR\left(\nu \frac{\pi_K + \delta_{\mathbf{D}^{\mathrm{K}}} + C_{\mathcal{P}}}{(1-\mathbf{D}^{\mathrm{Y}})(1-A)} - CRs_{rep}d - \Delta(\omega, r, d)\right) + (1-CR)\kappa(\cdot)\right]$
- $CR = \min(cr_0 + \frac{d(1-\mathbf{D}^{\mathrm{Y}})(1-A)}{\nu}, 1)$
- $r = \max\left\{0, r^* + i + \phi(i - i^*)\right\}$

B Initial values of the Model

The mentioned macroeconomic database gathers data from the World Bank, Penn University, the U.S. Bureau of Economic Analysis and the United Nations.

Symbol	Description	Value	Remarks/sources
CO_2^{AT}	CO_2-e conc. in the atmosphere layer	851 Gt C	DICE model, Nordhaus (2016)
CO_2^{UP}	CO_2-e concentration in the biosphere and upper ocean layer	460 Gt C	DICE model, Nordhaus (2016)
CO_2^{LO}	CO_2-e concentration in the deeper ocean layer	1,740 Gt C	DICE model, Nordhaus (2016)
d	Private debt ratio of the economy	1.53	Calibrated, macroeconomic database
E_{ind}	Industrial CO_2-e emissions	35.85 Gt CO_2-e	DICE model, Nordhaus (2016)
E_{land}	Exogenous land-use change CO_2-e emissions	2.6 Gt CO_2-e	DICE model, Nordhaus (2016)
F_{exo}	Exogenous radiative forcing	0.5 W/m^2	DICE model, Nordhaus (2016)
g_σ	Growth rate of the emission intensity of the economy	- 0.0152	DICE model, Nordhaus (2016)
p	Composite good price level	1	Normalization constant
p_{BS}	Backstop price level	547.22	DICE model, Nordhaus (2016), compound to 2016
n	Emissions reduction rate	0.03	DICE model, Nordhaus (2016)
N	Workforce of the economy in billions	4.83	Calibrated, macroeconomic database
NG	Total population in billions	7.35	Calibrated, macroeconomic database
T	Temperature in the atmosphere, biosphere and upper ocean layer	0.85 °C	DICE model, Nordhaus (2016)
T_0	Temperature in the deeper ocean layer	0.0068 °C	DICE model, Nordhaus (2016)
Y	Real world income in trillion 2015US$	59.7387	Calibrated, macroeconomic database
λ	Employment rate of the economy	0.675	Calibrated, macroeconomic database
ω	Wage share of the economy	0.578	Calibrated, macroeconomic database

C Calibration of the Model

The mentioned macroeconomic database gathers data from the World Bank, Penn University, the U.S. Bureau of Economic Analysis and the United Nations. For the sake of completeness, the value of the heat capacity of the atmosphere, biosphere, and upper ocean, C, has been adjusted according to Nordhaus and Sztorc (2013) whenever the value of the ECS, S, was different from its mean, so as to account for changes in the TCS whenever ECS varies.

Symbol	Description	Value	Remarks and sources
C	Heat capacity of the atmosphere, biosphere and upper ocean	1/.098 SI	DICE model, Nordhaus (2016), adjusted for a continuous framework
C_0	Heat capacity of the deeper ocean	3.52	DICE model, Nordhaus (2016), adjusted for a continuous framework
$C_{AT_{pind}}$	CO_2-e preindustrial concentration in the atmosphere layer	588 Gt C	DICE model, Nordhaus (2016)
$C_{LO_{pind}}$	CO_2-e preindustrial concentration in the deeper ocean layer	1,720 Gt C	DICE model, Nordhaus (2016)
CR_0	Constant of the credit rationing function, $\text{CR}(\cdot)$	0.17	Dafermos et al. (2017)
CR_1	Slope of the credit rationing function, $\text{CR}(\cdot)$	1	Adapted from Dafermos et al. (2017)
div_0	Constant of the dividend function, $\Delta(\cdot)$	0.051	Calibrated, macroeconomic database, more details available upon request
div_π	Slope of the dividend function, $\Delta(\cdot)$	.4729	Calibrated, macroeconomic database, more details available upon request
$[div_{\min}, div_{\max}]$	Range of the dividend function, $\Delta(\cdot)$	[0,1]	Selected among a range of reasonable values
$F_{2\times CO_2}$	Change in the radiative forcing resulting from a doubling of CO_2-e concentration w.r.t. to the pre-industrial period	3.681 W/m^2	DICE model, Nordhaus (2016)
F_{exo}^{start}	Initial value of the exogenous radiative forcing	0.5 W/m^2	DICE model, Nordhaus (2016)
F_{exo}^{end}	Value of the exogenous radiative forcing in 2100	1 W/m^2	DICE model, Nordhaus (2016)
f_K	Fraction of environmental damage allocated to the stock of capital	1/3	Dietz and Stern (2015)
i^*	Interest rate targeted by the monetary	0.02	Selected among a range of values that coincide with the mandate of most developed countries' Central Bank
P^N	Upper limit of the workforce dynamics in billions	7.056	Calibrated, macroeconomic database, more details available upon request
P_G^N	Upper limit of the total population dynamics in billions	12	Calibrated, macroeconomic database, more details available upon request
r^*	Long-term interest rate target of the economy	0.01	Selected among a range of reasonable values
s_{rep}	Fraction of the outstanding debt repaid yearly	0.1	Dafermos et al. (2017)
γ^*	Heat exchange coefficient between temperature layers	0.0176 SI	Nordhaus (2016), adjusted for a continuous framework
δ	Depreciation rate of capital	0.04	Inklaar and Timmer (2013)
$\delta_{E_{Land}}$	Growth rate of land-use change CO_2-e emissions	-0.022	DICE model, Nordhaus (2016), adjusted for a continuous framework
δ_{g_σ}	Variation rate of the growth of emission intensity	- 0.001	DICE model, Nordhaus (2016), adjusted for a continuous framework
$\delta_{p_{BS}}$	Exogenous growth rate of the back-stop technology price	- 0.005	DICE model, Nordhaus (2016), adjusted for a continuous framework
δ_N	Speed of growth of the workforce dynamics	0.0305	Calibrated, macroeconomic database, more details available upon request
$\delta_{N,G}$	Speed of growth of the total population dynamics	0.027	Empirically calibrated, macroeconomic database, more details available upon request
ζ_3	Damage function parameter	6.754	Weitzman (2011) and Dietz and Stern (2015)
η	Relaxation parameter of the inflation	0.192	Selected among a range of reasonable values
θ	Parameter of the abatement cost function	2.6	DICE model, Nordhaus (2016)
κ_0	Constant of the investment function, $\kappa(.)$	0.0397	Empirically estimated, macroeconomic database, more details available upon request
κ_1	Slope of the investment function, $\kappa(.)$	0.719	Empirically estimated, macroeconomic database, more details available upon request
$[\kappa_{min}, \kappa_{max}]$	Range of the investment function, $\kappa(.)$	[0,.3]	Selected among a range of reasonable values

Symbol	Description	Value	Remarks and sources
μ	Mark-up of prices over the average cost	1.875	Selected among a range of reasonable values
ν	Constant capital-to-output ratio	2.7	Inklaar and Timmer (2013)
π_1	Damage function parameter	0 /°C	DICE model, Nordhaus (2016), adjusted for a continuous framework
π_2	Damage function parameter	0.00236/°C^2	DICE model, Nordhaus (2016)
π_3	Damage function parameter in the Stern case	0.0000819/°C$^{\zeta_3}$	Dietz and Stern (2015)
ϕ	Parameter characterising the reactivity of the monetary policy	0.5	Taylor (1993)
ϕ_0	Constant of short-term Phillips curve, $\phi(.)$	-.292	Empirically calibrated, macroeconomic database, more details available upon request
ϕ_1	Slope of the short-term Phillips curve, $\phi(.)$	.469	Empirically calibrated, macroeconomic database, more details available upon request
Φ_{12}	Transfer coefficient for carbon from the atmosphere to the upper ocean/biosphere	0.024	DICE model, Nordhaus (2016), adjusted for a continuous framework
Φ_{23}	Transfer coefficient for carbon from the upper ocean/biosphere to the lower ocean	0.001	DICE model, Nordhaus (2016), adjusted for a continuous framework

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